Chapter 7

The concept of vibration isolation with variable stiffness with the use of magnetic elements

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Abstract
The aim of the work is to develop the concept of elements of the vibration isolation system using the magnetic field of solid magnets and to determine the influence of this field on the physico-mechanical parameters of the selected type of elastomer, which was subjected to static and dynamic load over a wide range of speed at room temperature. The article presents both theoretical foundations of the proposed solution and its experimental verification.

Keywords: vibration isolation system, solid magnets, elastomeric materials

1. Introduction

This paper shows a preliminary concept of vibration isolation systems, in which the use of magnetic elements in the elastomer structure is proposed. In addition, a pilot study of the physical-mechanical properties of elastomeric materials with magnets has been carried out and they can be used in the application to vibration isolation systems in the future. A discrete-continuous model of the vibration isolation system, both under kinematic and force excitation, consisting of an elastomer and a pair of neodymium magnets oriented uniquely with respect to each other, was developed. Their application, causes a change of both stiffness and damping in the vibration isolation system. This paper presents the results of preliminary tests of the elastomer and magnet pair system, which were performed at room temperature. As part of the work, it was also necessary to design a fixture for the testing machine that would allow safe testing for different temperatures in the climate chamber, as well as testing for different compositions of elastomers and magnets. The scope of this article covers:

1. development of a conceptual model of a vibration isolation system using constant field magnets,

2. development of a concept and design of a structural fixture for testing elastomeric components with permanent magnets in a testing machine,
3. development of testing methodology for elastomeric elements with permanent magnets,
4. preliminary results of elastomer testing with permanent magnets.

2. Physical and mathematical models of the vibration isolation system with elastomers and magnetic elements

The adopted physical model of the vibration isolation system with kinematic forcing, shown in Figure 1, consists of a protected object of mass m (1), an elastomer (2) in which permanent magnets are located (3) and where they facing each other with opposite poles. The elastomer (2) is forced kinematically according to the law of motion *y(x,t)*.

We treat the above model as a discrete-continuous system, which is described by a system of partial and ordinary differential equations, with boundary and initial conditions. The equations describing the vibrations of this conceptual vibration isolation system were derived assuming that there is no dependence of elastomer parameters on the magnetic field generated by permanent neodymium magnets. In these models, the magnets are facing each other with identical poles, which leads to the presence of a repulsive force.

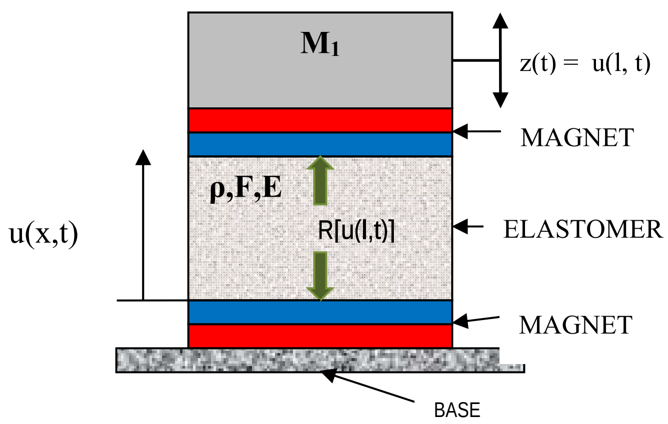

Figure 1. Schematic of the physical model of vibration isolation under kinematic forcing with a magnetic element

The differential equation describing the model with kinematic forcing (Figure 1), was derived assuming that the characteristics of the rubber is linear and has a continuous structure, then the vibration of the elastic element is described by a partial differential equation having the following form:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = a^2 \frac{\partial^2 u(x,t)}{\partial x^2}. \tag{1}$$

For relation (1) we introduce the following boundary conditions:

$$\begin{aligned} &(M_1 + m_1)\ddot{z} + EF\frac{\partial u(l,t)}{\partial x} = R[u(x,t)] \\ &u(0,t) = 0 \end{aligned} \tag{2}$$

where:

E – dynamic Young's modulus, *ń* – density of the rubber material, *F* – rubber cross-sectional area, M_1 – mass of the isolated object, m_1 – mass of the magnet, *R[u(x,t)]* – force between magnets as a function of distance change.

The relation describing the interaction of two cylindrical permanent magnets is very complex [1], but it can be reduced to a simplified formula of as follows:

$$\begin{aligned} R[u(x,t)] = A[&\frac{1}{u(x,t)^2} + \frac{1}{(u(x,t)+2h)^2} + \\ &- \frac{1}{(u(x,t)+h)^2}] \end{aligned} \tag{3}$$

where:

$A = \frac{\pi\mu}{4} M^2 r^4$ – constant [Nm²], *M* – magnetization [Am²/m³], r – radius of curvature of the magnet [m], *u(x,t)* – function of changing distance between magnets [m], h – magnet height [m], μ – magnetic permeability.

The relation (3) allows to determine the constant A based on the determined curve from the tests on the testing machine (Figure 2), the force F(x) can be determined for the position and type of the magnetic element of interest [2, 3].

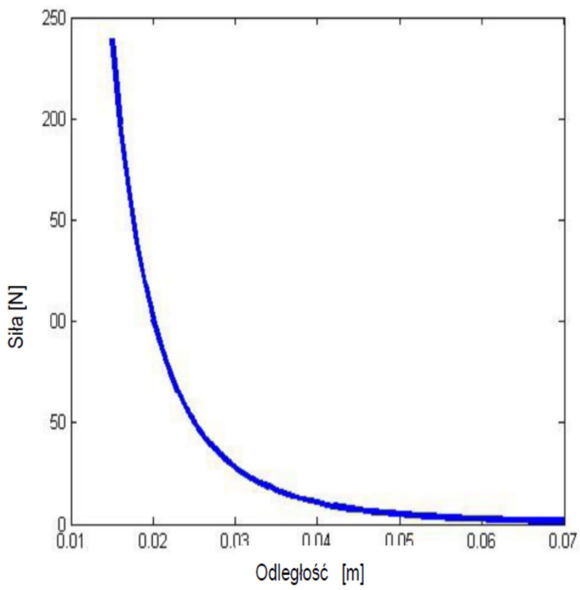

Figure 2. The distance dependence of the force for a magnet with diameter equal to 40 [mm] and height of 8 mm

Solution of the problem (1) with boundary conditions (2) is a nonlinear problem. It can be solved by linearizing relation (3), but assuming that the distance between magnets is significant, much larger than "l". Hence, practically the fastest and most convenient way to solve this problem is to use the FEM (Finite Element Method). However, in order to have input parameters for the model created by the FEM method, it is necessary to obtain these parameters by experimental investigation.

The study of the vibration isolation system with permanent magnets and rubber can be carried out by the method presented in [3], however, the problem of the possibility of the magnets affecting the elastomer parameters should be noted. Therefore, additional tests must be performed independently of the magnets themselves and the elastomer itself and then their combination to determine if such a phenomenon occurs. The problem required to be solved was the lack of a fixture that would allow us to perform tests for different compositions of magnets and elastomers. In order to do so, we made a dedicated fixture (Figure 3) for the Instron 8872 testing machine shown in Figure 4. Making a dedicated fixture was necessary in order to be able to perform the tests, the idea of which is shown in the block diagram shown in Figure 5.

Figure 4. Photo of the fixture

Figure 5. Instron 8872 testing machine

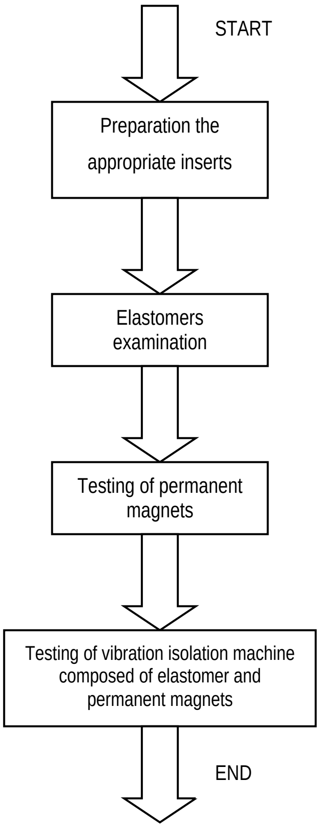

Figure 5. Block diagram of testing vibration isolator containing elastomer and pair of permanent magnets

3. Preliminary results of elastomer tests without and with magnets

Preliminary tests of the elastomer with a bulk density ρ = 699 kg/m^{-3}, porosity 38% and Shore A hardness 32°Sh, were carried out on the stand shown in Figure 4 at room temperature 20°C and for different strain rates. Parameters such as pore volume and sample hardness calculation were performed based on the methodology presented in [4], which was also used in the determination and calculation of elastomer parameters such as:

1. strain – ε_L,
2. modulus of elasticity for the linear section – E_L,
3. conventional modulus of elasticity for longitudinal section – $E_{L\text{-}20\%}$,
4. coefficient of damping (loss), stiffness for the linear section (k_L) and for the nonlinear section ($k_{L\text{-}20\%}$).

The results of these tests are presented in Table 1 and Table 2. Table 3 shows the results of calculations of the quasi-static and dynamic stiffness coefficient of the tested elastomer for a range of linear and nonlinear deformations as a function of the strain rate of the elastomer without and with magnets applied.

Table 1. Results of tests of conventional ε_L and conventional modulus of elasticity E_L as a function of the strain rate v of elastomer

System	ε_L [%]			E_L [MPa]		
	0.1 mm/s	a) 1.0 mm/s	b) 3.0 mm/s	c) 0.1 mm/s	d) 1.0 mm/s	e) 3.0 mm/s
Without magnets	14.05	12.8	14	0.44	0.47	0.50
With magnets	12.3	8.6	7.4	1.59	1.61	1.6

Table 2. Results of the conventional $E_{L-20\%}$ elastic modulus and damping factor (measure of dissipation) ψ as a function of strain rate v of the elastomer

System	$E_{L-20\%}$ [MPa]			[-]		
	f) 0.1 mm/s	g) 1.0 mm/s	h) 3.0 mm/s	i) 0.1 mm/s	j) 1.0 mm/s	k) 3.0 mm/s
Without magnets	0.76	0.78	0.82	0.22	0.29	0.27
With magnets	4.94	5.78	5.69	0.30	0.32	0.35

Table 3. Results of calculations of stiffness coefficient k_L for linear deformation range and $k_{L-20\%}$ for nonlinear deformation range as a function of strain rate v of elastomer

System	k_L [MN/m]			$k_{L-20\%}$ [MN/m]		
	l) 0.1 mm/s	m) 1.0 mm/s	n) 3.0 mm/s	o) 0.1 mm/s	p) 1.0 mm/s	q) 3.0 mm/s
Without magnets	0.76	0.78	0.82	0.22	0.29	0.27
With magnets	4.94	5.78	5.69	0.30	0.32	0.35

Figure 6 and Figure 7 show examples of hysteresis loops obtained by testing the elastomer without magnets and with magnets applied according to the scheme presented in Figure 1.

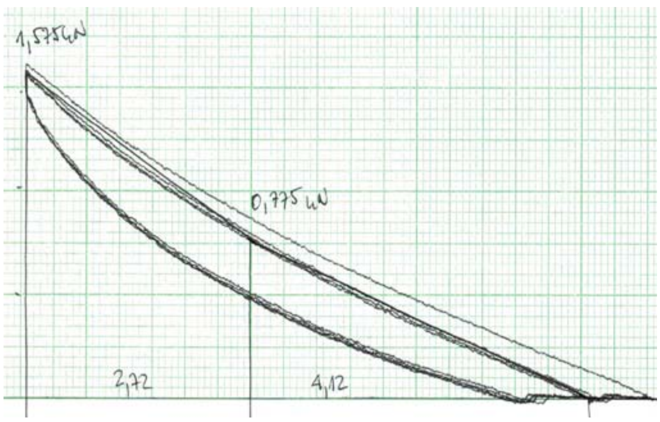

Figure 6. Hysteresis loop for elastomer without magnets at room temperature

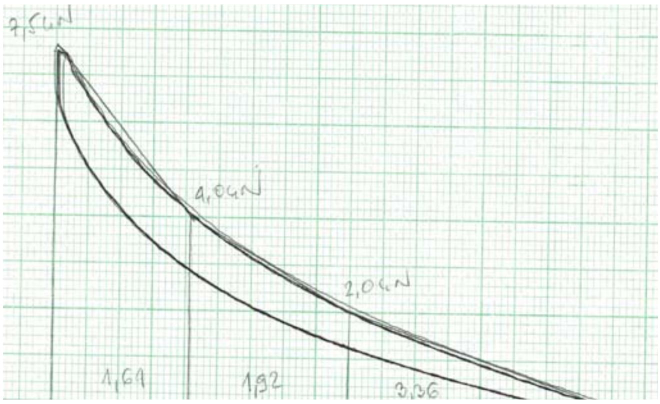

Figure 7. Hysteresis loop for elastomer with magnets at room temperature

The results were compared according to the relation:

$$y = \frac{x_z - x_{bez}}{x_{bez}} \cdot 100\%$$

where:

x_z – parameter of material with magnet, x_{bez} – parameter of material without magnet.

The results of the calculations are summarized in Table 4, Table 5 and Table 6.

Table 4. Comparison of results of tests of conventional ε_L and conventional modulus of elasticity E_L as a function of the strain rate v of elastomer

ε_L [%]			E_L [%]		
0.1 mm/s	1.0 mm/s	3.0 mm/s	0.1 mm/s	1.0 mm/s	3.0 mm/s
-12.46	-32.81	-47.89	261.36	242.55	220.00

Table 5. Comparison of results of the conventional $E_{L-20\%}$ elastic modulus and damping factor (measure of dissipation) ψ as a function of strain rate v of the elastomer

$E_{L-20\%}$ [%]			ψ [%]		
0.1 mm/s	1.0 mm/s	3.0 mm/s	0.1 mm/s	1.0 mm/s	3.0 mm/s
550.00	641.02	593.90	36.36	10.34	29.63

Table 6. Comparison of results of calculations of stiffness coefficient k_L for linear deformation range and $k_{L-20\%}$ for nonlinear deformation range as a function of strain rate v of elastomer

k_L [%]			$k_{L-20\%}$ [%]		
0.1 mm/s	1.0 mm/s	3.0 mm/s	0.1 mm/s	1.0 mm/s	3.0 mm/s
262.07	238.71	218.18	550.00	641.46	386.11

By analyzing the above results, there is a significant increase in the stiffness of the system as well as the damping ratio, however, it is significantly less than the stiffness.

4. Summary

This paper shows the methodology for testing elastomers with permanent magnets. It was also shown that the use of permanent magnets significantly affects the change in stiffness and damping of elastomers. The use of permanent magnets in the construction of vibration isolators is justified in some cases as it allows the stiffness to be controlled, i.e. the mechanical system can be tuned in such a way as to minimize the dynamic effects on the protected object. The analytical solution is too time-consuming, if possible, so at present it is proposed to use numerical methods to approximate the solution of this problem. Therefore, it is necessary to carry out experimental studies with this type of vibration isolation systems, the results of which will be the input parameters for the calculation of numerical models and will allow simulations of the dynamics of this type of isolation systems.

References

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