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# Computerized Method for Work Space Optimization in Conditions of Static Work

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The aim of this research was to develop a theoretical method for the ergonomic optimization of the work space of the upper limb. This method is based on a model of the upper extremity with 7 degrees of freedom. It consists of 3 rigid elements modeling the arm, forearm, and hand and 34 upper extremity muscles. The trunk is considered immobile. The shoulder joint is modeled as a rotating kinematics pair of third class, the elbow and wrist joints—of fourth class. The minimum sum of muscle force moments in the joints and soft saturation muscle cooperation criterion were used as merit criteria. The developed method makes it possible to effectively solve, in a defined work space, the task of work space optimization.

work space optimization computer modeling upper extremity

## 1. INTRODUCTION

Nowadays, heavy physical work is done by machines. As a consequence, more and more frequently human performs the so-called light static work, which is often connected with high repeatability of movements or with motionless posture for many hours a day. As a result of local fatigue with low but long-lasting effort, there is an untypical exertion of

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other muscles that can take over the work of the fatigued muscles. This causes untypical musculoskeletal load, mostly of the spine, and it causes damage and disorders of the musculoskeletal system (Andersson, 1984; Westgaard & Aaras, 1985). Thus, this kind of work, even with quite low muscular load (low energetic and overload effects), is rather strenuous and causes negative consequences for the musculoskeletal system (Hagberg, & Wegman, 1987; Roman, Bugajska, & Konarska, 1996). However, very often—mostly due to economical reasons—this kind of work is not eliminated.

Improvement and protection against negative outcomes of this kind of work can be searched in work optimization of factors influencing musculoskeletal load. The three basic factors that influence that load are location of the body, external force, and the frequency of repetition of a given task or duration of work. One of the most important aspects is optimization of the area in which physical work is performed by upper limbs. Work space optimization is connected with choosing from a determined three-dimensional work space, subspaces in which work-related effort, measured in some defined way, is lowest. Until now, mostly experimental methods, which are very expensive and time consuming, have been used for work space optimization. A theoretical computerized method would thus be very useful. Theoretical methods for musculoskeletal load assessment are usually multibody-type models of the human body. In those models, net muscle moments that the worker has to exert in the joints so as to balance the weight of his or her body and the external force are defined (Chaffin & Andersson, 1991). Those methods are advanced, however, the last stage, which would allow them to be used for work space optimization, is missing. As work space optimization is closely connected with upper extremity location, to adequately design a modern work place, one needs a method that allows to estimate the effort of the muscle group performing given work as a function of positioning the point where the force should be applied in the work space. It is possible to find an optimum configuration of the body, for which the sum of the absolute values of net muscle moments in the joints will be lowest, and in the neighborhood of that configuration to determine a sub-work space in which work will be performed with little muscle effort. The development of a theoretical method that makes work space optimization possibile would provide a good experimental and practical tool for designing work stands.

The aim of this study was to develop, on the basis of the upper limb

model, a new theoretical method that can be used for ergonomic work space optimization.

### 2. PHYSICAL MODEL

The physical model consists of a kinematics chain of the upper extremity, muscles, and work space. The physical model of the upper extremity is open and has 7 degrees of freedom. It consists of three rigid elements modeling the arm, forearm, and hand with constant, for a given participant, dimensions and masses (Figure 1).



Figure 1. Physical model of kinematics chain of the upper extremity: (a) front view, (b) shoulder joint. Notes.  $X_0$ ,  $Y_0$ ,  $Z_0$ —global coordinates connected with an immobile trunk;  $X_{j_i}$   $Y_{j_i}$   $Z_{j_i}$ —local coordinate systems for j = 1, 2, ..., 7, according to Denavit-Hartenberg principles; coordinates  $Z_0$ ,  $Z_6$ ,  $Z_7$  are perpendicular to the figure plane.

The trunk was considered immobile. The shoulder joint was modeled as a rotating kinematics pair of third class (3 degrees of freedom) and the elbow and wrist joints—of fourth class (2 degrees of freedom). This model is a high simplification of reality as the human upper extremity

has about 27–30 degrees of freedom and 7 degrees is a minimum, which allows the location of the extremity in space and a grip. A reduction of the degrees of freedom was necessary to simplify the model, which allows computer calculations.

The model takes into account all basic movements of the upper limb, defined in relation to the frontal plane—abduction/adduction, sagittal plane—flexion/extension, and pronation/supination defined as

Muscle Name (Latin)	Cross-Section (mm <sup>2</sup> )
latissimus dorsi pars vertebralis	300
latissimus dorsi pars costalis	300
latissimus dorsi pars iliaca	300
pectoralis major pars abdominalis	350
pectoralis major pars sternocostalis	300
pectoralis major pars clavicularis	300
deltoideus pars clavicularis	1090
deltoideus pars acromialis	1090
deltoideus pars spinalis	1090
supraspinatus	100
infraspinatus	250
teres major	300
teres minor	300
suscapularis	400
coracobrachialis	70
biceps brachii caput breve	260
biceps brachii caput longum l	260
biceps brachii caput longum II	260
brachioradialis	70
triceps brachii caput longum	530
triceps brachii caput mediale	750
triceps brachii caput laterale	350
brachialis	730
anconeus	300
supinator	180
pronator teres	180
extensor carpi radialis longus	50
extensor carpi radialis brevis	100
extensor digitorum communis	200
extensor carpi ulnaris	290
flexor carpi ulnaris	350
flexor carpi radialis	300
flexor policis longus	300
flexor digitorum sublimis	480

TABLE 1. Muscles Taken into Account in the Computer Model and Their Cross-Sections

rotation round the axis of the limb. Coordinates of the points of muscles attachment to bones were taken from Seireg and Arvicar (1989). They are defined according to the local Denavit-Hartenberg coordinate system (Denavit & Hartenberg, 1955). The center of rotation in the shoulder joint is the center of the global coordinate system. Thirty-four muscles of the upper extremity were modeled. The cross-sections of the individual muscles taken in this model are presented in Table 1. It has been assumed that the maximum force  $F_{max}$  developed by the muscles is a product of a cross-section by allowable muscle tension (1 MPa for each muscle).

The shape of the work space was adopted as a segment of a sphere inside the area of maximal upper limb reach. This space is defined by the following parameters (Figure 2):

- the polar coordinates of the sphere center in relation to the global center of coordinates (center of rotation in the shoulder joint)— $X_p$ ,  $Y_p$ ,  $Z_p$ ;
- the internal radius of the sphere segment—r;
- the external radius of the sphere segment—R;
- the horizontal angles of the sphere— $\alpha_1$ ,  $\alpha_2$ ;
- the vertical angles of the sphere— $\beta_1$ ,  $\beta_2$ .



**Figure 2. Work space defined in the model.** Notes.  $X_{\rho_1} Y_{\rho_1} Z_{\rho}$ —polar coordinates of the sphere center in relation to the global center of coordinates; *r*—internal radius of the sphere segment; *R*—external radius of the sphere segment;  $\alpha_1$ ,  $\alpha_2$ —horizontal angles of the sphere;  $\beta_1$ ,  $\beta_1$ —vertical angles of the sphere.

## **3. MATHEMATICAL MODEL**

The physical model of the upper limb was formalized in an analytical form into a mathematical model. The analytical formula describes the fact that in a chosen point in the work space, the kinematics chain of the upper limb stays in static balance under its own load, forces of muscles, and external force. Computer software CAMIR (Rzymkowski, 1988) comprising a program for symbolic operations (it converts algebraic formulas) was used for mathematical calculations. In this way, seven equations were generated (one equation for each degree of freedom).

$$\sum_{i=1}^{34} F_i r_{ij} + M_{aj} + M_{zj} = 0$$
 (1)

where  $F_i$ —force generated by the *i*-th muscle (i = 1, ..., 34);  $r_{ij}$ —the arm of force exertion in relation to the axis of rotation of the *j*-th degree of freedom (j = 1, 2, ..., 7);  $M_{aj}$ —the contribution of gravity forces in the equation of force moments in relation to the *j*-th axis of rotation;  $M_{zj}$ —the contribution of the external force in the equation of force moments in relation.

It was accepted that those equations were obligatory for the angles in the joints that were in the range defined by

$$q_{j\min} \leqslant q_j \leqslant q_{j\max} \tag{2}$$

where  $q_j$ —the angle of rotation in the joint in accordance with the *j*-th degree of freedom. Values of the angles  $q_{j \min}$  and  $q_{j \max}$  define the maximum range of the physiological angles of the movements in the joints. Muscle forces must be within the following values:

$$0 \leqslant F_i \leqslant F_{i\max} \leqslant f(l) \tag{3}$$

where

$$F_{i\max} = S_i \times 2 \text{ MPa} \tag{4}$$

and  $S_t$ —the cross-section of the *i*-th muscle in m<sup>2</sup>; 2 MPa—maximum allowable muscle tension; f(l)—a dimensionless function expressing the dependence between maximum muscle force from the muscle length:

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$$f(l) = \sin \frac{(l - l_{\min}) \times \Pi}{l_{\max} - l_{\min}}$$
(5)

where  $l_{\min}$ ,  $l_{\max}$ —minimum and maximum muscle length; it was accepted that  $l_{\min} = 0.5 \ l_0$  and  $l_{\max} = 1.5 \ l_0$ , where  $l_0$ —muscle length when the limb location is in the middle between extreme limb locations.

In the mathematical model of the upper limb, there are seven equations with 34 unknown values (muscle forces), which makes the mathematical task statically indeterminable (excess of muscles in relation to the degrees of freedom). The solution of this problem, also called the solution of muscle contribution, is usually searched with the assumption that the nervous system controls muscles according to some merit criterion. In this study, a merit criterion of "soft saturation" was used (Equation 6). It had been proved that the results of calculations made according to this criterion—the best criterion of all—are in step with the experimental results (Siemieński, 1992):

$$\sum_{i=1}^{34} 1 - \sqrt{1 - \frac{F_i}{F_{i\max}(l)}} = \min$$
 (6)

where  $F_i$  and  $F_{i \max}$  are the same as in Equations 3 and 4.

It can be stated that from the aforementioned mathematical formulas. Equation 1 is a mathematical model of muscle cooperation, inequalities 2 and 3 together with 4 and 5 express the constraint condition, and Equation 6 is the merit criterion of the first optimization problem. The solution of this problem for a given external force leads to the calculation of muscular forces in one upper extremity location described by a set of angle values  $(q_1, ..., q_7)$  describing limb location. Achieving the purpose of the optimization problem, however, requires finding an optimum upper limb location (calculating an optimal set of angles  $q_1, \ldots, q_7$ ), one in which the muscular load will be lowest. Thus, to complete the task, it is necessary to carry out an additional optimization process, in which the merit function will be a formal formula for a value proportional to muscle effort in static work conditions. On the basis of the results of other studies (Ayoub, 1994; Seireg & Arvicar, 1989), it was accepted that this merit function is expressed as the sum of modules of muscle forces in relation to the axis of rotation in the joints, which must be developed by the muscles in the arm and the elbow and wrist joints to balance the limb's own weight. The form of the second merit criterion is

$$\sum_{j=1}^{7} \sum_{i=1}^{34} |F_i r_{ij}| = \min$$
 (7)

where  $F_i$  and  $r_{ij}$  are the same as in Equation 1.

### 4. NUMERICAL OPTIMIZATION OF THE WORK SPACE

The described mathematical model, together with the constraint condition and double optimization, was transformed into a computer simulation model, which makes it possible to find an effective solution of the task of work space optimization. Figure 3 presents a diagram of the optimization process conducted with this system.

It is the task of the user to define the work space for a given work task. The user also gives parameters connected with the dimensions and



Figure 3. Diagram of the optimization process of the upper limb work space.

masses of upper limb segments, and the value and direction of the external force.

Double optimization is performed: "external" using the Monte Carlo method (Goliński, 1974) and the second merit criterion (Equation 7), and "internal" with the first merit criterion (Equation 6) and the gradient method of optimization. Such a solution is the result of preliminary studies, which showed that internal optimization can be conducted by using the classic gradient method. Using the gradient method for external optimization causes many local minima of function and does not always give reasonable solutions (Kędzior, Roman, & Rzymkowski, 1993c). To overcome this problem, for the purpose of outside optimization, the Monte Carlo method is needed.

During the optimization process, the computer program performs the following steps in succession.

- It randomly generates a set of seven physiologically admissible values of the angles of rotation in the joints.
- It determines the location of the palm mass center, checks whether the mass center is within the work space, and—if it is—the program performs the next move, if not—it goes back to the generation of a new set of joint angles.
- It determines and stores in memory all 34 muscle forces assuming that the limb is in static balance; in order to solve this problem a merit criterion (Equation 6) is used.
- It determines and stores in memory the value of the objective function (Equation 7) and presents the results in a graphical form.

The system presents solutions to the optimization task obtained in subsequent cycles of calculations in a graphical mode. Work space is divided into 625 small subspaces (Kędzior, Roman, & Rzymkowski, 1993a). In each subspace, the calculated value of the merit function is marked in color. Moreover, information concerning the minimum and maximum values of the merit function and the relevant angles in the joints is available. The user follows the optimization process and, on the basis of current results, decides about the definition of the big subspaces of the work space that should be examined closer, or accepts the results and finishes the optimization process.

Obtaining a solution usually requires conducting even several thousand cycles of calculations, because the calculation is based on the double

merit criterion. Calculations conducted for different input data showed that the program for work space optimization works correctly (Kędzior, Roman, & Rzymkowski, 1993b).

### 5. EXAMPLE

The following example illustrates the method. The aim is to determine a subspace in a given work space, where the effort of the muscles driving the human arm is lowest. It has been assumed that the work is done with the right upper extremity using a 0.5-kg tool held in hand and that the work must be performed with eye-sight control. A physical model of the human upper limb is driven by 22 muscles. The actual number of muscles is 34, but in order to overcome the problem of the too small computer Random Access Memory (RAM), this number has



Figure 4. Graphical illustration of optimization results: (a) vertical cross-section of the work space by a plane marked on Figure b as A-A, (b) horizontal cross-section of the work space marked on Figure a as B-B. *Notes.* +—optimum solution.

been reduced in such a way that some muscle groups have been substituted by one muscle representative.

The case was calculated for the following values of the parameters defining the work space (Figure 2):  $X_p = 18 \text{ cm}$ ,  $Y_p = -20 \text{ cm}$ ,  $Z_p = -18 \text{ cm}$ , r = 30, R = 60,  $\alpha_1 = 45^\circ$ ,  $\alpha_2 = 45^\circ$ ,  $\beta_1 = 50^\circ$ ,  $\beta_2 = 50^\circ$ , where  $X_p$ ,  $Y_p$ ,  $Z_p$ —the polar coordinates of the sphere center in relation to the global center of coordinates; *r*—the internal radius of the sphere segment; *R*—the external radius of the sphere segment;  $\alpha_1$  and  $\alpha_2$ —the horizontal angles of the sphere;  $\beta_1$  and  $\beta_2$ —the vertical angles of the sphere.

The dimensions of the human upper extremity were taken as the arm is 29 cm, the forearm is 23 cm. The masses of the links were calculated from the Zatsiorsky formula (Zatsiorsky, Aruin, & Sieluyanov, 1981) and their values were as follows: the arm is 1.9 kg, the forearm is 1.2 kg, the hand is 0.4 kg. Figure 4 presents the solution in a graphical form.

### 6. SUMMARY

The presented example shows that the computer program works correctly. This method is general and it can be used to determine optimum subspaces of the work space for various physical jobs. However, experimental verification is necessary. In the simulation model of the upper extremity, work space optimization is conducted by comparing muscular effort for a participant for different limb locations and for a given external force. On the basis of Equation 7, optimum values, which can be verified experimentally, are values of muscle forces. For verification purposes for different limb positions and different external force, forces in muscles and the sum of forces in all 34 muscles can be calculated. Those values should be compared with the parameters assessing muscular tension and fatigue obtained in experimental studies.

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