International Journal of Occupational Safety and Ergonomics (*JOSE*) 2007, Vol. 13, No. 4, 381–389

A Power Transformer as a Source of Noise

Wiktor Marek Zawieska

Central Institute of Labour Protection – National Research Institute, Poland

This article presents selected results of analyses and simulations carried out as part of research performed at the Central Institute of Labor Protection – the National Research Institute (CIOP-PIB) in connection with the development of a system for active reduction of noise emitted by high power electricity transformers. This analysis covers the transformer as a source of noise as well as a mathematical description of the phenomenon of radiation of vibroacoustic energy through a transformer enclosure modeled as a vibrating rectangular plate. Also described is an acoustic model of the transformer in the form of an array of loudspeakers.

power transformer noise reduction active methods transformer modeling rectangular plate vibrations transformer noise vibroacoustic analysis

1. INTRODUCTION

Power transformers are a source of low-frequency noise, with the most important spectrum components, in view of the noise level, in the frequency range below 400 Hz. The noise is of stationary nature with a spectrum with clearly visible components for frequencies which are a multiple of the double frequency of the power grid. An example of a power transformer noise spectrum with a power of 630 kVA is shown in Figure 1.

The level of noise emitted by transformers depends to a large extent on their power, size and load [1, 2, 3, 4]. In practice it can be assumed that the flow of vibroacoustic energy in an oil-

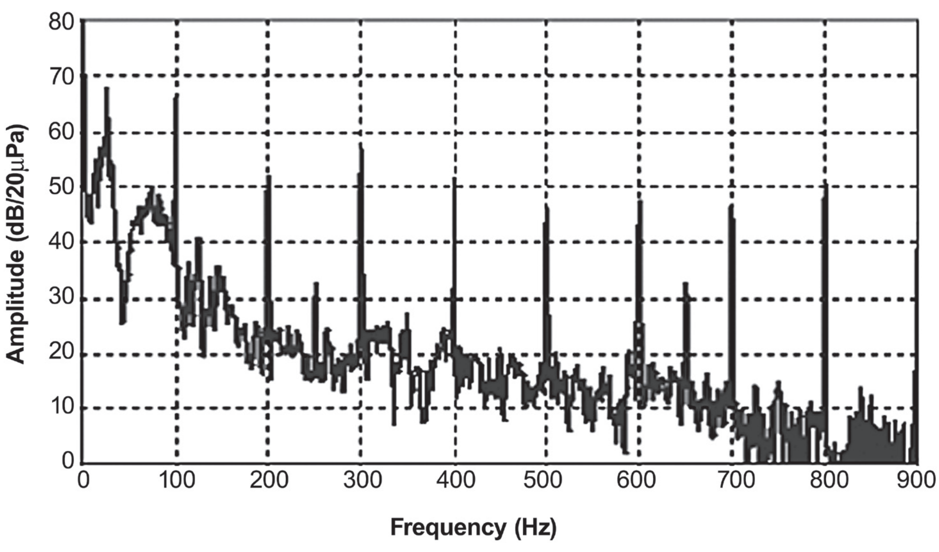

Figure 1. An example of a spectrum of power transformer noise

Correspondence and requests for offprints should be sent to Wiktor Marek Zawieska, Department of Vibroacoustic Hazards, Central Institute of Labour Protection – National Research Institute, 00-701 Warszawa, Czerniakowska 16, Poland. E-mail: <mazaw@ciop.pl>.

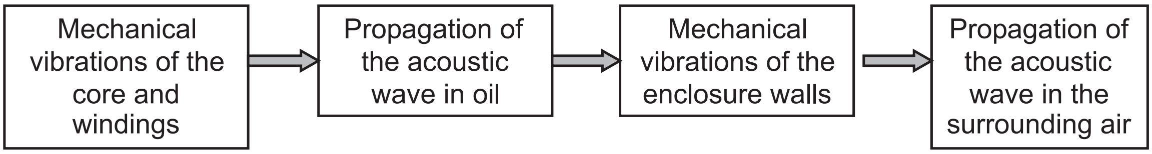

Figure 2. A diagram of vibroacoustic energy flow in an oil-immersed transformer.

immersed transformer, the most common power transformer type, is as shown in Figure 2.

Research carried out, among others, at the Central Institute of Labor Protection – the National Research Institute (CIOP-PIB) indicates that in transformers working in real-life conditions the sources of vibroacoustic energy are as follows: the vibrating core, magnetostriction, vibrating winding, vibrating structural components, etc. This energy is carried over mainly to the transformer enclosure. Therefore, it can be assumed that the vibroacoustic energy emitted (radiated) into the surroundings is created first of all as a result of enclosure vibrations. Transformer enclosures, due to material, design and operational considerations, come in different shapes. For the purposes pertaining to research on the possibility of using active methods in order to control transformer noise it can be assumed that the enclosure is cubicoid.

Research on using active methods to control noise emitted by transformers has been carried out in Poland and abroad for several dozen years. As part of research carried out at CIOP-PIB, a system for active control of transformer noise (ACTN) was developed and verified in real-life conditions after laboratory tests. Each stage of the development of ACTN showed that the mathematical modeling stage as well as laboratory investigations based on the transformer acoustic model are of key importance for the selection of design solutions and the attained parameters of designed systems and the final cost. The results of research on modeling a transformer as a source of noise are presented in a later part of the article.

2. MODELING A TRANSFORMER AS A SOURCE OF NOISE

The development of a mathematical model of a power transformer as a source of noise is primarily justified for two reasons: (a) a description of acoustic wave emissions of a power transformer makes it possible to perform various computer calculations and simulations (Matlab® suite version 5 was used); (b) on the basis of the developed mathematical description of the phenomenon of emission of vibroacoustic energy and the calculations and simulations performed, it is possible to develop an acoustic model of a power transformer as a source of noise. This model can be used successfully in laboratory investigations of the use of active methods of noise control in power transformers. Performing such research on an actual transformer is troublesome (e.g., due to restricted access) and often dangerous.

In the development of a computational and mathematical model of a power transformer the following assumptions were made.

- An actual power transformer is the reference point.
- The nature of the phenomena which are the main sources of noise in power transformers indicate that the acoustic power is emitted in very narrow bands around even-numbered harmonics of the power grid frequency. Therefore, it can be assumed that the transformer emits acoustic energy into the environment only at frequencies which are even-numbered harmonics of the standard power grid. Because in Poland the frequency is 50 Hz, transformer noise can be described with vibrating surface sounds with harmonic frequencies of 100 and 200 Hz. Here, we ignore all sources of noise of a different nature

(such as fans) and the deviations from the standard frequency of 50 Hz, which can occur in natural conditions.

- Acoustic energy is emitted only through the vibrating walls of the enclosure (an oil vat) of the transformer.
- In the simulation process a rectangular plate was adopted as a model of the transformer enclosure wall. This assumption makes it possible to describe mathematically the phenomenon of emission of vibroacoustic energy through a vibrating, rectangular plate (see, e.g., Rdzanek, Rdzanek Engel, et al. [5]). The vibrating plate can be treated as a surface made of small components that emit an acoustic wave. These components must be small enough for the assumption that all points of such a component vibrate with the same amplitude and phase to be true. This means that the dimensions of the component must be much smaller than the length of the shortest structural wave of the plate under consideration. Using this approach, the vibrating plate can be replaced with an array of loudspeakers playing the role of the aforementioned components. Through individual control of the amplitude and phase of each loudspeaker in the array it is possible to obtain very different distributions of the sound field.

3. SOUND EMISSION THROUGH THE ENCLOSURE (PLATE) OF THE TRANSFORMER

Adopting the aforementioned assumptions, the amplitudes and phases of individual components of which the plate is comprised can be determined from the plate vibration equation. In order to obtain all data required in the simulation process a theoretical analysis was performed of the vibroacoustic properties of a flat, rectangular plate with articulated support and with fixed mounting. Following are the results of analyses, which have been presented in more detail in, e.g., Zawieska [6], Leissa [7], Meirovitch [8], Wallace [9], Davies [10].

Let us assume that a flat plate with dimensions of $a \times b$ satisfies the homogeneous equation of free vibration, which can be formulated for mode (m, n) as follows:

$$(k_{mn}^{-4}\nabla^4 - 1)W_{mn}(x, y) = 0, \qquad (1)$$

where x, y—co-ordinates of a point located on the plate surface; $k_{mn}^2 = \pi^2[(m/a)^2 + (n/b)^2$—a structural wave number pertaining to the mode of free vibration (m, n); $\nabla^4 = [(\partial^2/\partial x^2) + (\partial^2/\partial y^2)]^2$—a biharmonic operator in Cartesian co-ordinates.

The form of free vibration of the plate for a pair of modal numbers of $m, n = 1, 2, 3, \ldots$ can be written down as

$$W_{mn}(x, y) = A_{mn}\sin(m\pi x/a)\sin(n\pi y/b), \qquad (2)$$

where a, b—geometric size of the plate.

The A_{mn} constant was calculated from the $\int_S W_{mn}^2 \mathrm{d}S = S$ normalization condition, obtaining $A_{mn} = 2$.

The boundary conditions of the plate are as follows:

$$\begin{aligned} &W(x, y)\,|_{x=\pm a/2} = 0, \quad \frac{\partial^2}{\partial x^2}W(x, y)\,|_{x=\pm a/2} = 0, \\ &W(x, y)\,|_{y=\pm b/2} = 0, \quad \frac{\partial^2}{\partial y^2}W(x, y)\,|_{y=\pm b/2} = 0, \end{aligned} \qquad (3)$$

where $W(x, y)$ is the function describing the transversal transfer of the plate points. The natural frequencies of the vibrating system are as follows:

$$\omega_{mn} = k_{mn}^2\sqrt{D/\rho h}, \qquad (4)$$

where $m, n = 1, 2, 3, \ldots$; ρ—plate density, h—plate thickness, D—plate rigidity.

The acoustic potential of the surface source in the Fraunhofer zone can be written down as [11, 12]

$$\begin{aligned} &\phi(\vec{r},\ t) = \phi(R,\ \vartheta,\ \varphi,\ t) \\ &= \frac{e^{[i(\omega t - kR)]}}{2\pi R}\int_{S_0} v(r_0)e^{[ikr_0\cos(\vec{r},\vec{r}_0)]}dS_0 \end{aligned} \qquad (5)$$

where $\cos(\vec{r}, \vec{r}_0) = \sin\vartheta\cos(\varphi - \varphi_0)$; $\vec{r} = (R, \vartheta, \varphi)$—field point radial vector in spherical co-ordinates; $\vec{r}_0 = (x_0, y_0)$—plate surface point radial vector in Cartesian co-ordinates (Figure 3).

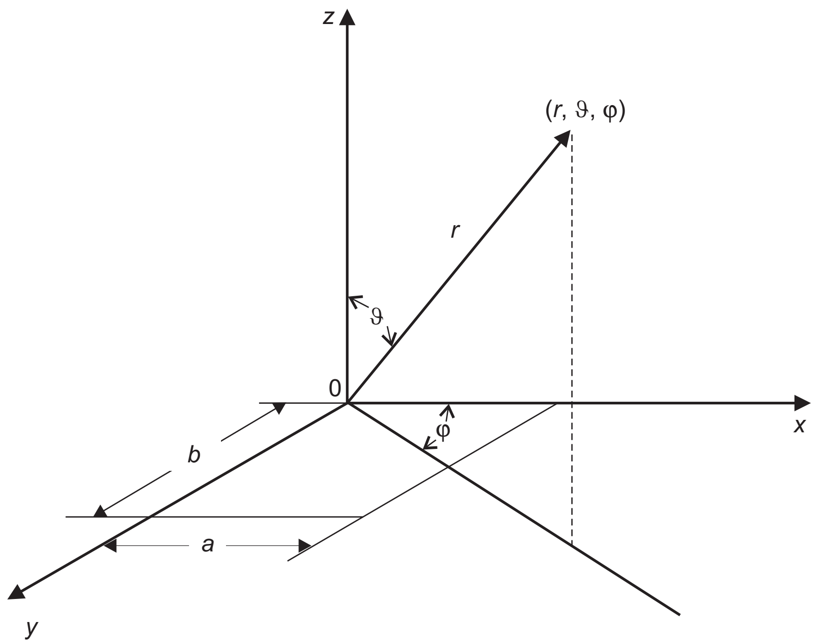

Figure 3. A rectangular plate in Cartesian and (*x*, *y*, *z*) spherical co-ordinates (*r*, θ, φ).

In the event of processes harmonically variable in time the following relationships can be written down for the distribution of vibration speed of an acoustic particle and the sound radiation:

$$v(\vec{r},t) = \rho_0 \frac{\partial}{\partial t} w(\vec{r},t) = i\omega w(\vec{r},t),$$
$$w(\vec{r},t) = W(\vec{r})\mathrm{e}(i\omega t), \qquad (6)$$
$$p(\vec{r},t) = \rho_0 \frac{\partial}{\partial t} \phi(\vec{r},t) = i\omega\phi(\vec{r},t),$$

for any field point described with the radial vector of $\vec{r}$. The sound radiation pressure in the Fraunhofer zone can be written down as

$$p(R,\vartheta,\varphi,t) = -\rho_0\omega^2 \frac{\mathrm{e}^{\left[i(\omega t - kR)\right]}}{2\pi R}$$
$$\bullet \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} W(x_0,y_0)\exp[ik\sin\vartheta(x_0\cos\varphi \qquad (7)$$
$$+ y_0\sin\varphi)]\mathrm{d}x_0\mathrm{d}y_0 .$$

The sound pressure amplitude $p_{mn}(R, \theta, \varphi)$ based on Equation 7 and the conversions which are not discussed here can be written down as an elementary formula:

$$\mathrm{P}_{mn}(R,\vartheta,\varphi) = -\rho_0 c \frac{ke^{-ikR}}{R}\omega_{mn}A_{mn}\frac{ke^{-ikR}}{R}\frac{u_1u_2}{2\pi}$$
$$= 2\rho_0 c\omega_{mn}A_{mn}\frac{kab}{\pi^3 nm}\frac{\mathrm{e}^{-\mathrm{ikR}}}{R} \qquad (8)$$
$$\frac{\psi_m(\beta_a/2)}{1-(\beta_a/m\pi)^2}\frac{\psi_n(\beta_b/2)}{1-(\beta_b/n\pi)^2}.$$

The $\mathrm{P}_{mn}(\theta, \varphi)$ modal index of directivity is defined as the value of sound pressure module from Equation 8, standardized by the value of this module along the (θ_0, φ_0) direction, along which this quantity takes the maximum proper value in the Fraunhofer zone:

$$\mathrm{P}_{mn}(\vartheta,\varphi) = \frac{\left|p_{mn}(R,\vartheta,\varphi)\right|}{\max\limits_{\vartheta,\varphi}\left|p_{mn}(R,\vartheta,\varphi)\right|}. \qquad (9)$$

The numerical analysis of the modal index of directivity of sound radiation through the tested acoustic system was performed on the basis of Equations 8 and 9. Directivity of radiation for several symmetrical modes $m = n$ is presented in Figure 4a. The shape of curves shown in it indicates that the radiated sound pressure reaches

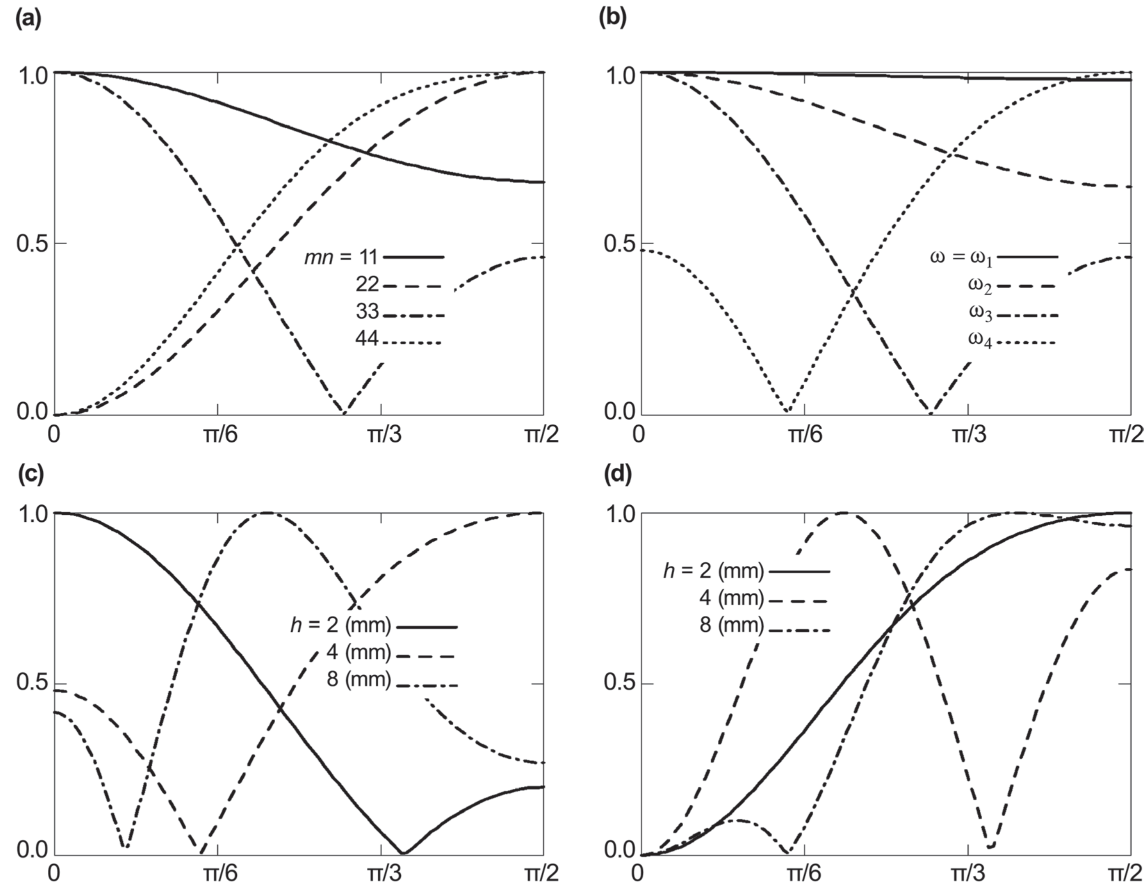

Figure 4. The modal index of radiation directivity for the selected four forms of free vibration of a rectangular plate with free articulated support for different values of plate parameters. *Notes.* a—$m = n = 1, 2, 3, 4$; $\omega = \omega_{33}$; $\varphi = \pi/2$; $h = 4$ mm; b—$m = n = 3$; $\omega = \omega_{11}, \ldots, \omega_{44}$; $\varphi = \pi/2$; $h = 4$ mm; c—$m = n = 3$; $\omega = \omega_{44}$; $\varphi = \pi/2$; $h = (2, 4, 8)$ mm; d—$m = n = 4$; $\omega = \omega_{44}$; $\varphi = \pi/2$; $h = (2, 4, 8)$ mm.

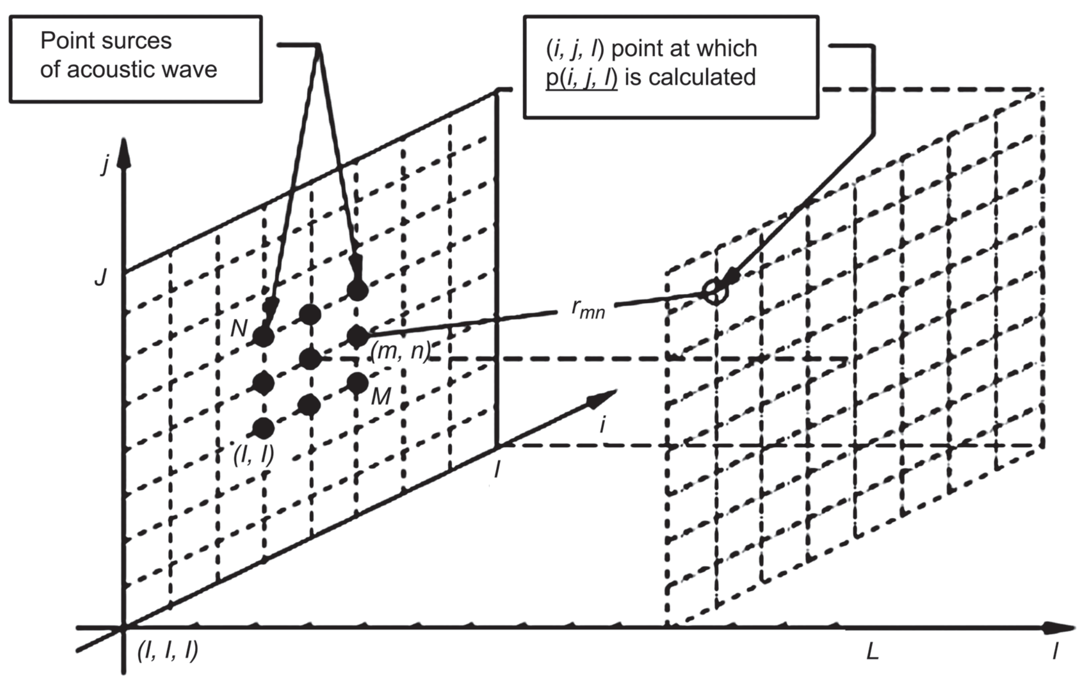

Figure 5. Loudspeakers arranged in the spatial co-ordinates system (*i*, *j*, *l*).

the maximum value along the main direction of the $\theta = 0$ plate for modes with odd numbers $m = n = 1, 3, \ldots$ and along a direction other than the main $\theta \neq 0$ direction for modes with even numbers $m = n = 2, 4, \ldots$.

Calculations were performed for the following values of parameters of the vibroacoustic system containing a vibrating rectangular plate: a homogeneous steel plate with density $\rho = 7700$ kg/m^3 and dimensions $a \times b$ ($a = 1.04$ m, $b = 0.52$ m), plate thickness $h = \{2, 4, 8\} \times 10^{-3}$ m, Poisson's ratio for steel $v = 0.3$, acoustic wave propagation speed in air $c = 340$ m/s, air density $\rho_0 = 1293$ kg/m^3, Young's modulus for steel $E = 205.0 \times 10^9$ N/m^2.

4. ACOUSTIC MODEL OF A POWER TRANSFORMER

As mentioned earlier the vibrating plate can be simulated with an acoustic system comprised of an array of loudspeakers, each of which can be treated as a source of a spherical wave (the analysis is performed for the far field of the Fraunhofer zone).

In order to perform simulations and computer calculations, the problem subject to analysis must be digitized in time and space. As regards the steady state under consideration, time variability can be disregarded in the calculations. Figure 5 shows the arrangement of an array loudspeakers with dimensions of (M, N) treated as point sources in the co-ordinate system (i, j, l).

The calculations take into account a digitized space with dimensions of (I, J, L). The spatial grid is cubical, with a side of d. The acoustic pressure in a complex form at point P(i, j, l), originating from an array of sources with dimensions of (M, N) is expressed with the following formula [13]:

$$p(i, j, l) = \left[j\omega\rho_0 \sum_{m=1}^{M} \sum_{n=1}^{N} \left(\frac{A_{mn}}{r_{mn}} e^{j(\varphi_{mn} - kr_{mn})} \right) \right] \bullet e^{j\omega} \tag{10}$$

for $i = 1, 2, \ldots, I$; $j = 1, 2, \ldots, J$; $l = 2, \ldots, L$; where

$$r_{mn} = d \bullet \sqrt{(i - m)^6 + (j - n)^2 + l^2}, \tag{11}$$

and d—resolution (side of the digitization grid), m.

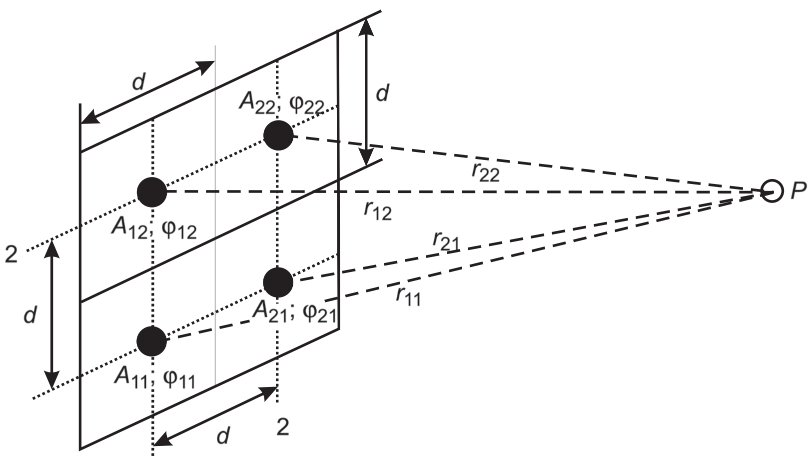

Figure 6. A model of a plate (enclosure wall) as an array comprised of four sources.

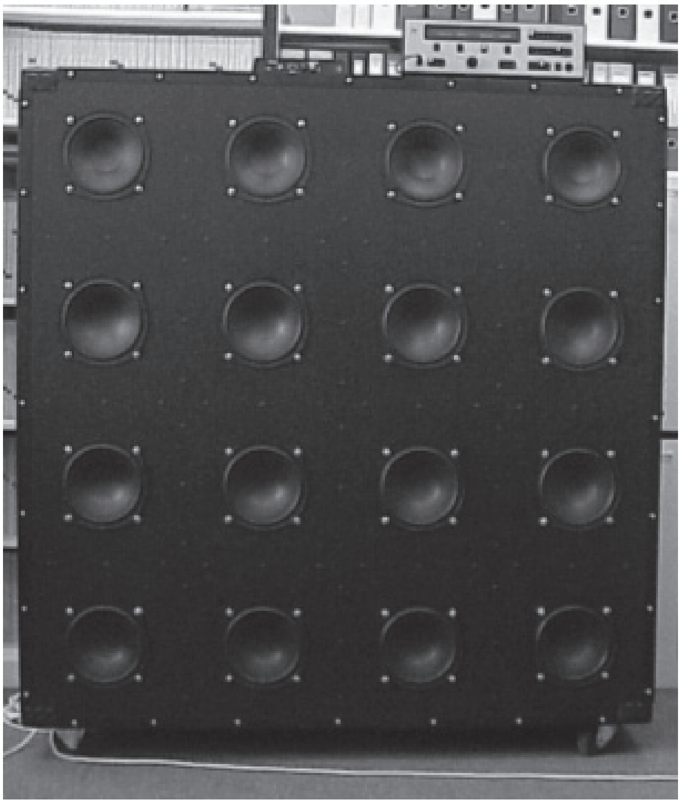

Figure 7. An acoustic model of a power transformer.

For the far field condition in the case under consideration to be satisfied, the following relationship must be fulfilled:

$$l >> kd^2. \quad (12)$$

The plate model in the form of an array comprised of four point sources is presented in Figure 6, and the acoustic model of a power transformer developed on the basis of the presented approach is shown in Figure 7. For the results of laboratory studies using the model of a power transformer see Zawieska [14].

5. SIMULATING STUDIES

The results of sample simulations of acoustic wave emission by a plate simulated in such a way are presented as pressure curves in Figure 8. Simulations were performed for two different source arrangements using Matlab® suite version 5.

Column A in Figure 8 shows the results for an arrangement of four sources placed on vertices of a square with sides of 0.75 m; two sources at opposite corners have phases shifted by π relative to the other two. The volume velocity of all sources was equal to 0.05 m^3/s. Column B shows the results for an analogous arrangement of sources, with the phase matched in all sources. Column C shows the scale of tones, in dB/20 μPa, for the presented charts.

Row 1 presents diagrams of the level of acoustic pressure complex modulus in the *OXY* plane at a distance of 2.25 m ($10\ kd^2$) from the source plane. Row 2 illustrates diagrams of the level of complex modulus of acoustic pressure in the *OXY* plane at a distance of 10 m from the source plane. Row 3 shows diagrams of the level of complex modulus of acoustic pressure in the *OXZ* plane horizontally (fixed co-ordinate y) of the first pair of sources.

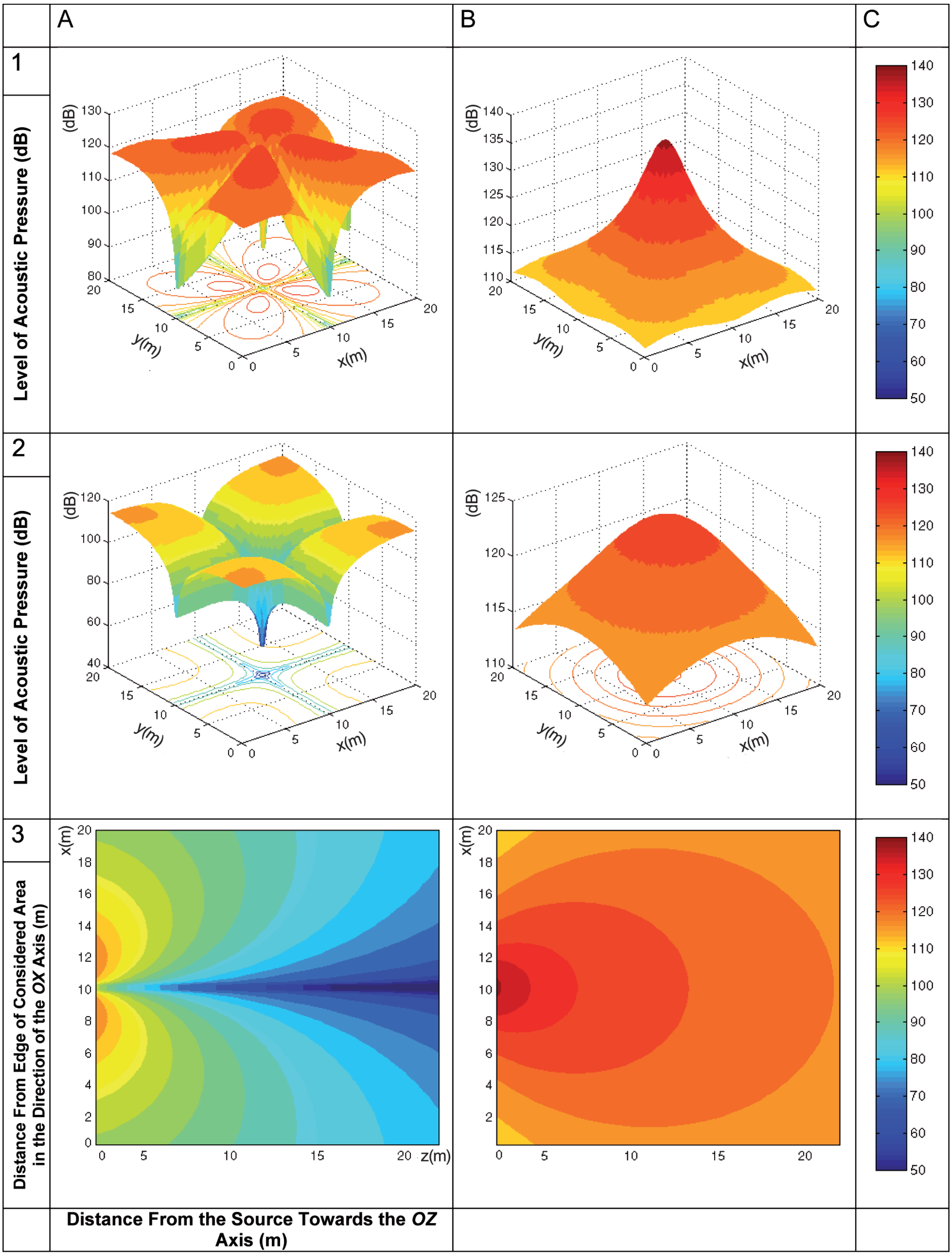

Figure 8. Results of simulation calculations of noise control in a transformer simulated by an array of loudspeakers.

6. CONCLUSIONS

The research and numerical simulations discussed in this paper showed that a power transformer as a source of noise can be simulated with good approximation as an array of point sources. With specific geometric conditions satisfied, these sources can be put into effect (in a certain frequency range) with appropriately selected loudspeakers. By controlling the amplitude and phase of each loudspeaker, the desired directional characteristics of the source can be obtained.

The mathematical simulation performed and the actual acoustic model of the transformer in the form of an array of loudspeakers made it possible to perform several simulations and laboratory investigations pertaining to the use of active methods for the control of noise emitted by transformers. Particularly valuable are the possibilities open by the use of developed models in designing active systems for use in actual conditions first of all during the phase of tuning the circuit controlling operation of the system and initial adjustment of the developed system in its final form.

REFERENCES

1. Byczkowska-Lipińska L. Analiza wpływu zjawisk elektrodynamicznych na postacie drgań i zagrożenia mechaniczne uzwojeń transformatorów. Łódź, Poland: Politechnika Łódzka; 1992.
2. Byczkowska-Lipińska L. Zjawiska wibroakustyczne w transformatorach energetycznych i ich wpływ na zagrożenie środowiska naturalnego. In: Akustyka w technice, medycynie i kulturze. Kraków, Poland: Komitet Akustyki PAN; 1999. p. 191–4.
3. Byczkowska-Lipińska L, Gryglewicz-Kacerka W, Młotkowski A. Modelling of free vibrations of a cylindrical coil. J Tech Phys. 1996;37(2):131–6.
4. Jezierski E. Transformatory. Warszawa, Poland: WNT; 1983.
5. Rdzanek WP, Rdzanek WJ, Engel Z, Szemela K. The modal low frequency noise of an elastically supported circular plate. International Journal of Occupational Safety and Ergonomics (JOSE). 2007;13(2):147–57.
6. Zawieska WM. Wybrane zagadnienia aktywnej redukcji hałasu na przykładzie transformatorów. Warszawa, Poland: Central Institute for Labour Protection – National Research Institute; 2007.
7. Leissa AW. Vibration of plates (NASA SP-160). Washington, DC, USA: U.S. Government Printing Office; 1969.
8. Meirovitch L. Analytical methods in vibration. New York, NY, USA: Macmillan; 1967.
9. Wallace CE. Radiation resistance of a rectangular panel. J Acoust Soc Am. 1972; 51(3):946–52.
10. Davies HG. Acoustic radiation from fluid loaded rectangular plates (Technical Report No. 71476-12). Cambridge, MA, USA: Massachusetts Institute of Technology; 1969.
11. Rdzanek WP Jr, Zawieska WM. Vibroacoustic analysis of a simply supported rectangular plate of a power transformer casing. Arch Acoust. 2003;28(2):117–25.
12. Malecki I. Physical foundations of technical acoustics. Oxford, UK: Pergamon Press; 1969.
13. Engel Z, Kowal J. Sterowanie procesami wibroakustycznymi. Kraków, Poland: Wydawnictwa AGH; 1995.
14. Zawieska WM. The active noise control issues related to the noise generated by the power transformers. Mechanics (AGH, Kraków). 2005;24(2):155–61. Retrieved November 28, 2007, from: http://journals.bg.agh.edu.pl/MECHANICS/2005-02/mech19.pdf